

## Section 4.1 Graphing Linear Equations in One Variable

**Example:** Write the equations of the horizontal line and the vertical line that pass through the point  $(2, 1)$ .

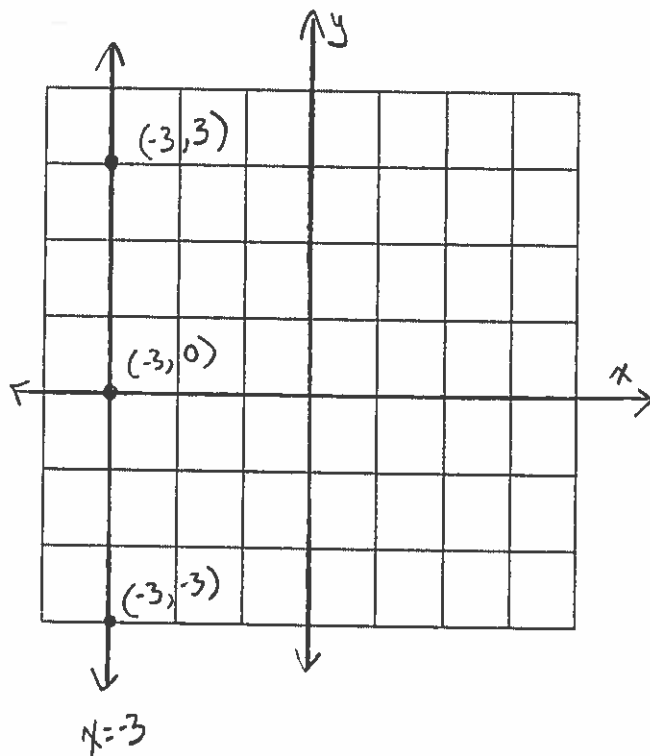
The  $x$ -coordinate of  $(2, 1)$  is 2. Because all of the points on the vertical line with  $(2, 1)$  will also have an  $x$ -coordinate of 2, the equation of the vertical line is  $x = 2$ . The  $y$ -coordinate of  $(2, 1)$  is 1. Because all of the points on the horizontal line with  $(2, 1)$  will also have a  $y$ -coordinate of 1, the equation of the horizontal line is  $y = 1$ .

**Answer:** The horizontal line is  $y = 1$  and the vertical line is  $x = 2$ .

**Example:** Sketch the graph of the line  $x = -3$  labeling three points.

The line  $x = -3$  is the set of all points that have  $x$ -coordinates of  $-3$ . Pick three such points. Examples could be  $(-3, 3)$ ,  $(-3, 0)$ , and  $(-3, -3)$ . When connected, these three points will form a vertical line.

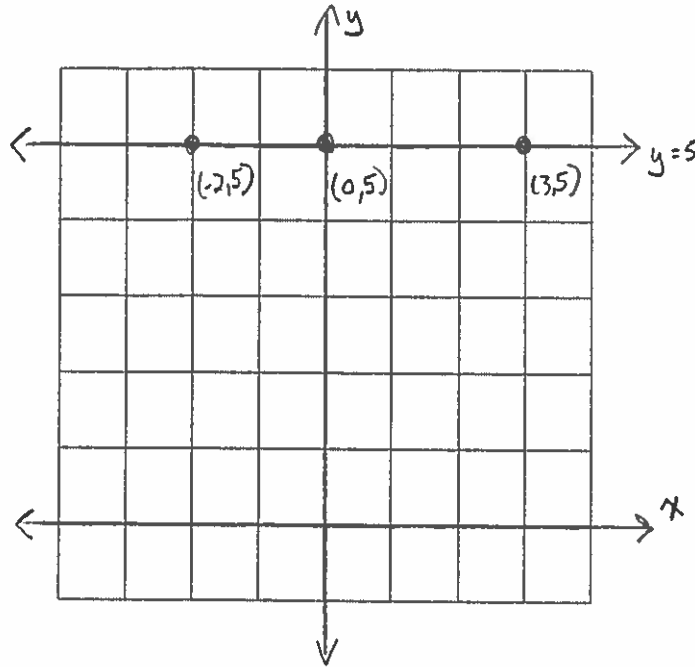
**Answer:**



**Example:** Sketch the graph of the line  $y = 5$  labeling three points.

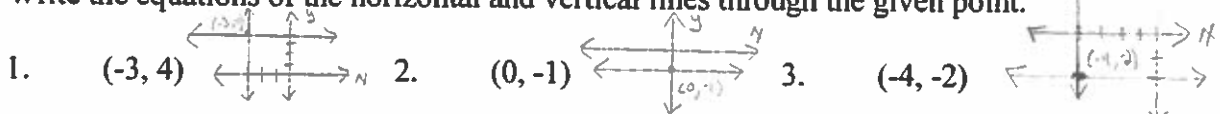
The line  $y = 5$  is the set of all points that have  $y$ -coordinates of 5. Pick three such points. Examples could be  $(-2, 5)$ ,  $(0, 5)$ , and  $(3, 5)$ . When connected, these three points will form a horizontal line.

**Answer:**



**Practice**

Write the equations of the horizontal and vertical lines through the given point.



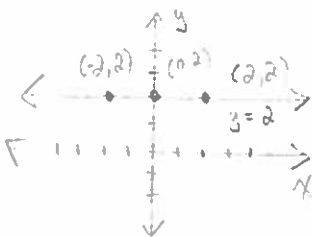
Horizontal Line:  $y = 4$       Horizontal Line:  $y = -1$       Horizontal Line:  $y = -2$

Vertical Line:  $x = -3$       Vertical Line:  $x = 0$       Vertical Line:  $x = -4$

Use a ruler and sketch the graph of the following lines labeling three points each.

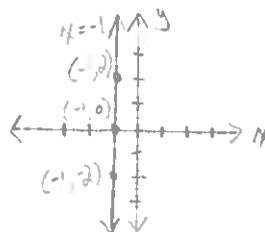
4.  $y = 2$

Domain $x$	Range $y = 2$	Solution $(x, y)$
-2	2	$(-2, 2)$
0	2	$(0, 2)$
2	2	$(2, 2)$



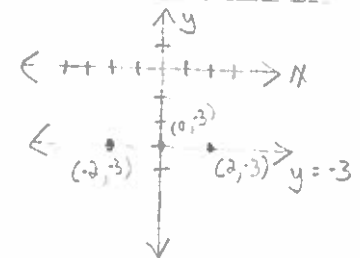
5.  $x = -1$

Domain $x = -1$	Range $y$	Solution $(x, y)$
-1	-2	$(-1, -2)$
-1	0	$(-1, 0)$
-1	2	$(-1, 2)$



6.  $y = -3$

Domain $x$	Range $y = -3$	Solution $(x, y)$
-2	-3	$(-2, -3)$
0	-3	$(0, -3)$
2	-3	$(2, -3)$



## Section 4.2 Graphing Linear Equations in Two Variables Using a Table

- Procedure:**
1. Solve the equation for  $y$ .
  2. Make a table of three  $x$ -values.
  3. Plot and label.

**Example:** Graph the equation  $6x - 2y = 4$ .

- Solution:**
1. Solve for the equation for  $y$ .

$$6x - 2y = 4$$

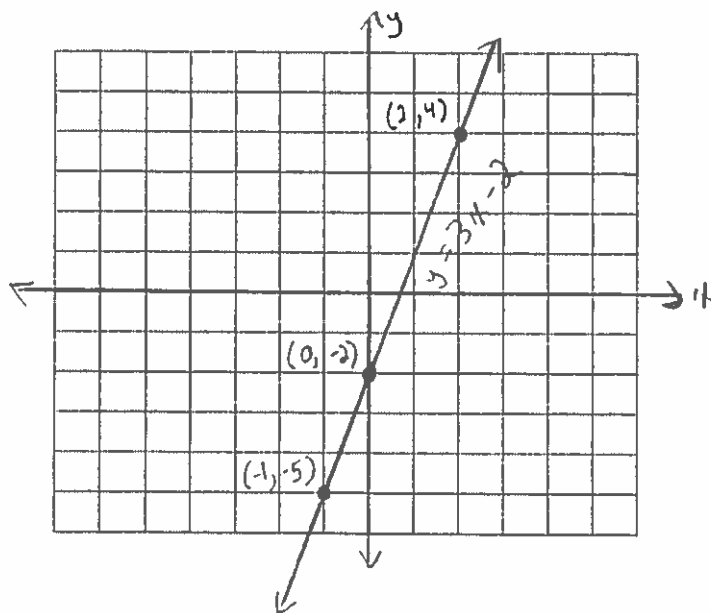
$$-2y = -6x + 4$$

$$y = 3x - 2$$

2. Make a table of three  $x$ -values. (Pick values around the origin.)

<i>Domain</i> $x$	<i>Range</i> $y = 3x - 2$	<i>Solution</i> $(x, y)$
-1	$y = 3(-1) - 2$ $y = -3 - 2$ $y = -5$	$(-1, -5)$
0	$y = 3(0) - 2$ $y = 0 - 2$ $y = -2$	$(0, -2)$
2	$y = 3(2) - 2$ $y = 6 - 2$ $y = 4$	$(2, 4)$

3. Plot and label.

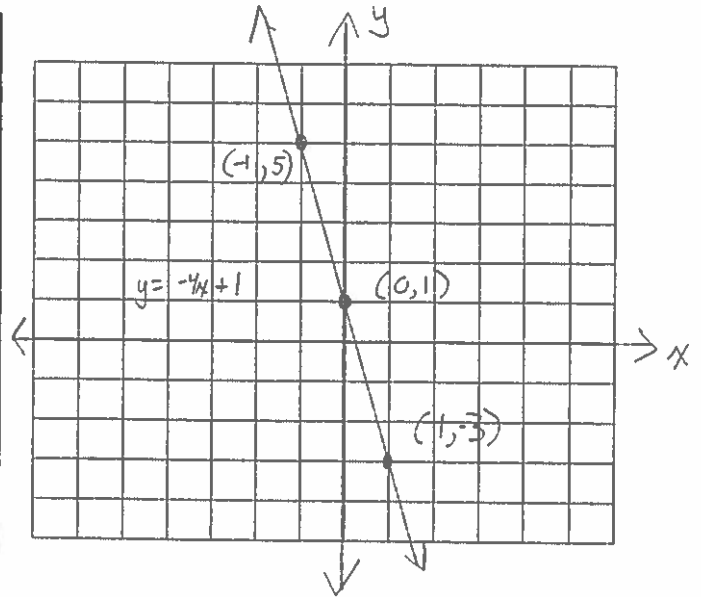


Practice for 4.2 Use a table of values to graph each equation. Follow the three steps.

1.  $4x + y = 1$

$$y = -4x + 1$$

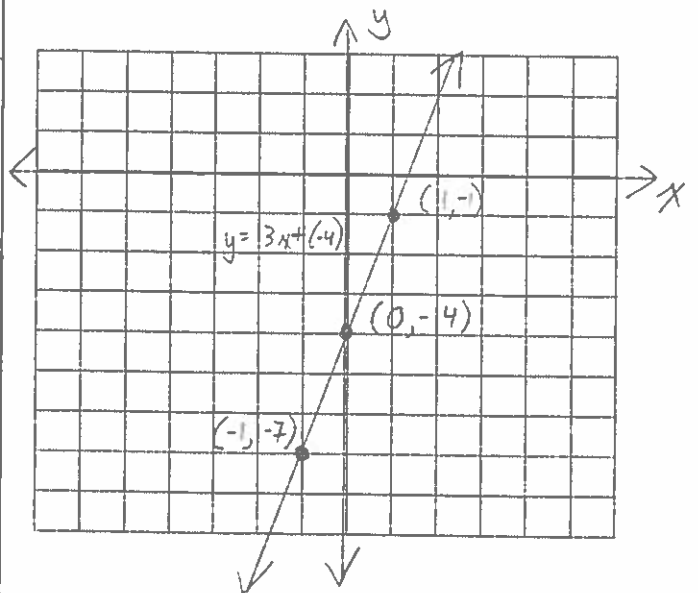
Domain	Range	Solution
$x$	$y = -4x + 1$	$(x, y)$
-1	$y = -4(-1) + 1$ $y = 4 + 1$ $y = 5$	$(-1, 5)$
0	$y = -4(0) + 1$ $y = 0 + 1$ $y = 1$	$(0, 1)$
1	$y = -4(1) + 1$ $y = -4 + 1$ $y = -3$	$(1, -3)$



2.  $9x - 3y = 12$

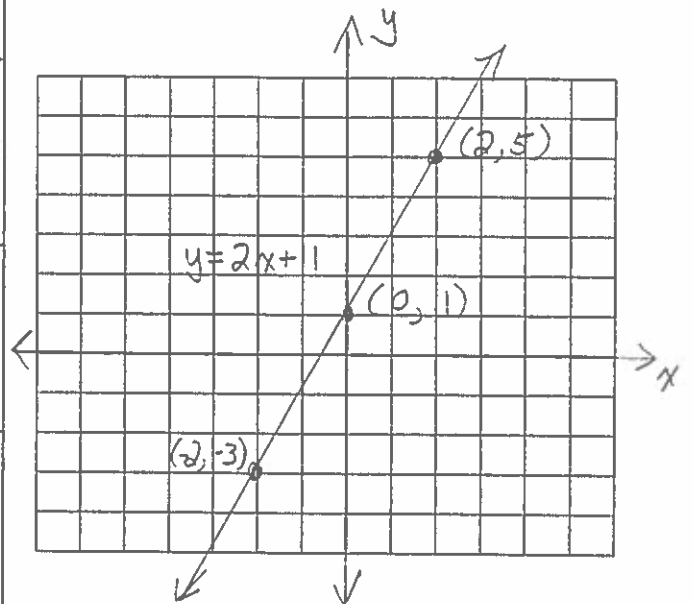
$$\begin{aligned} -3y &= -9x + 12 \\ y &= 3x + (-4) \end{aligned}$$

Domain	Range	Solution
$x$	$y = 3x + (-4)$	$(x, y)$
-1	$y = 3(-1) + (-4)$ $y = -3 + (-4)$ $y = -7$	$(-1, -7)$
0	$y = 3(0) + (-4)$ $y = 0 + (-4)$ $y = -4$	$(0, -4)$
1	$y = 3(1) + (-4)$ $y = 3 + (-4)$ $y = -1$	$(1, -1)$



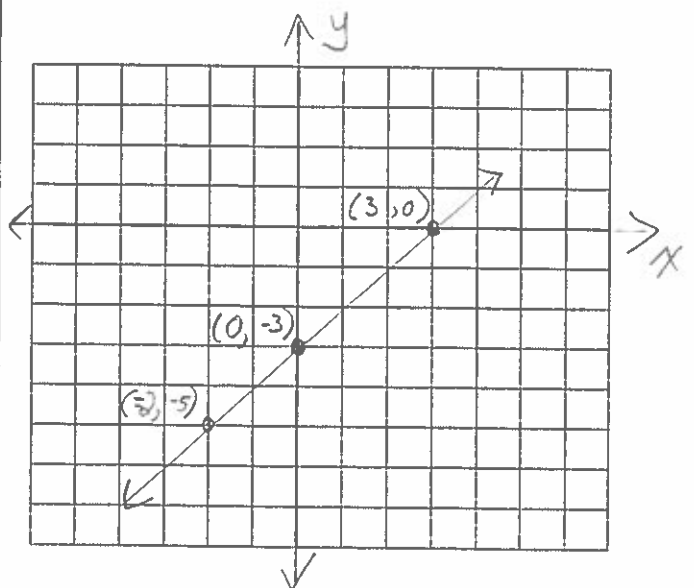
3.  $-4x + 2y = 2$   
 $2y = 4x + 2$   
 $y = 2x + 1$

Domain	Range	Solution
$x$	$y = 2x + 1$	$(x, y)$
-2	$y = 2(-2) + 1$ $y = -4 + 1$ $y = -3$	$(-2, -3)$
0	$y = 2(0) + 1$ $y = 0 + 1$ $y = 1$	$(0, 1)$
2	$y = 2(2) + 1$ $y = 4 + 1$ $y = 5$	$(2, 5)$



4.  $5x - 5y = 15$   
 $-5y = -5x + 15$   
 $y = x + (-3)$

Domain	Range	Solution
$x$	$y = x + (-3)$	$(x, y)$
-2	$y = -2 + (-3)$ $y = -5$	$(-2, -5)$
0	$y = 0 + (-3)$ $y = -3$	$(0, -3)$
3	$y = 3 + (-3)$ $y = 0$	$(3, 0)$



Section 4.3 Using Intercepts to Sketch the Graph of a Linear Equation

Example: Sketch the graph of  $2x + 3y = 6$  using intercepts.

The  $x$ -intercept is the point where the line crosses the  $x$ -axis. Every point on the  $x$ -axis has a  $y$ -coordinate of 0. Substitute  $y = 0$  into the original equation to find the  $x$ -intercept.

$$2x + 3(0) = 6$$

$$2x = 6$$

$$x = 3$$

The  $x$ -intercept is  $(3, 0)$ .

The  $y$ -intercept is the point where the line crosses the  $y$ -axis. Every point on the  $y$ -axis has an  $x$ -coordinate of 0. Substitute  $x = 0$  into the original equation to find the  $y$ -intercept.

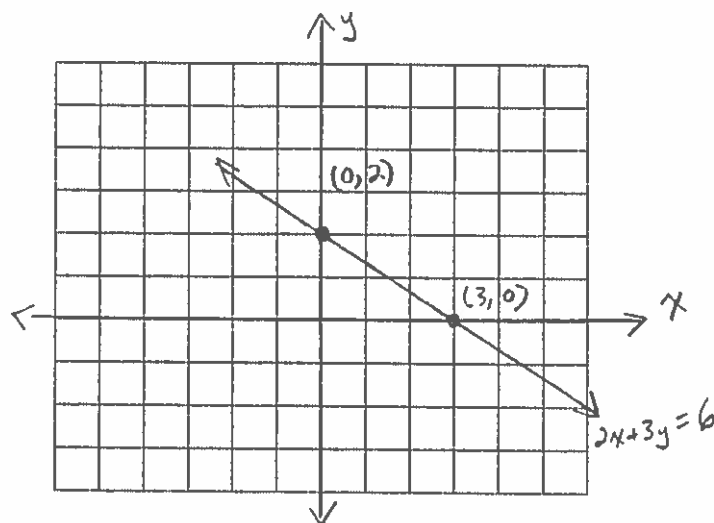
$$2(0) + 3y = 6$$

$$3y = 6$$

$$y = 2$$

The  $y$ -intercept is  $(0, 2)$ .

Plot and label.



Example: Sketch the graph of  $-x + 3y = 9$  using intercepts.

$$-x + 3(0) = 9$$

$$-x = 9$$

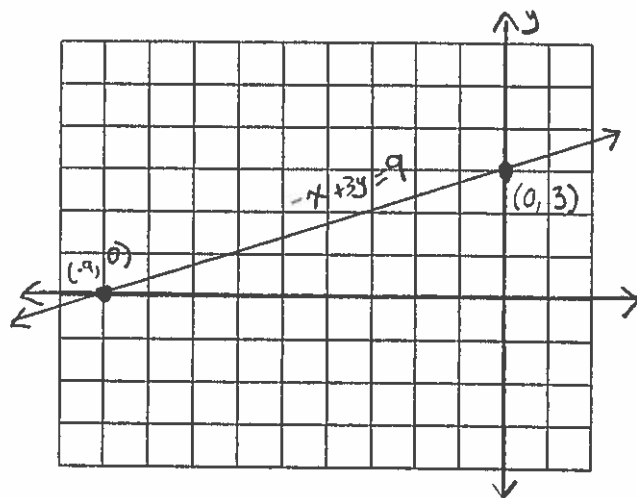
$$x = -9$$

$$-(0) + 3y = 9$$

$$3y = 9$$

$$y = 3$$

The x-intercept is  $(-9, 0)$  and the y-intercept is  $(0, 3)$



Practice for 4.3 Use intercepts to sketch the graphs of each linear equation.

1.  $-2x + 3y = -6$

x-int:  $y = 0$

$$-2x = -6$$

$$x = 3$$

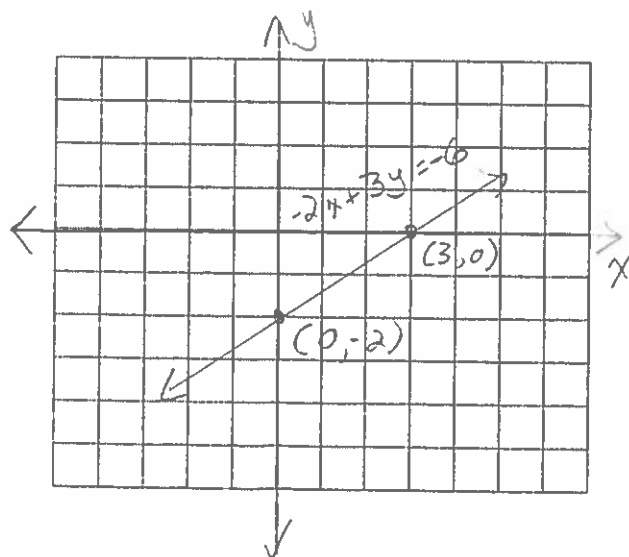
$$(3, 0)$$

y-int:  $x = 0$

$$3y = -6$$

$$y = -2$$

$$(0, -2)$$



2.  $2x + y = -4$

x-int:  $y = 0$

$$2x = -4$$

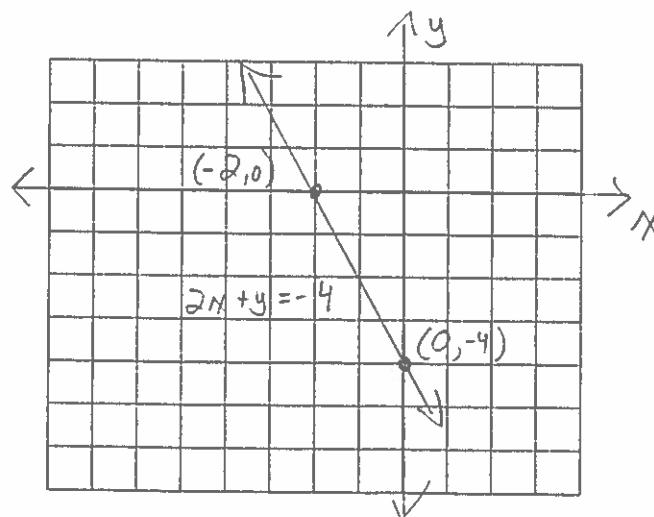
$$x = -2$$

$$(-2, 0)$$

y-int:  $x = 0$

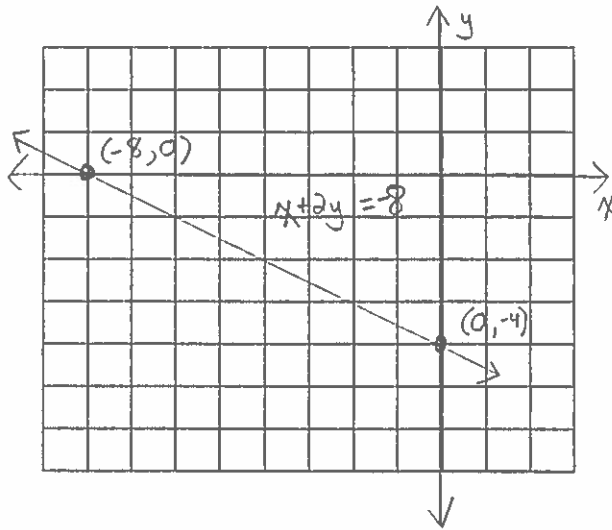
$$y = -4$$

$$(0, -4)$$



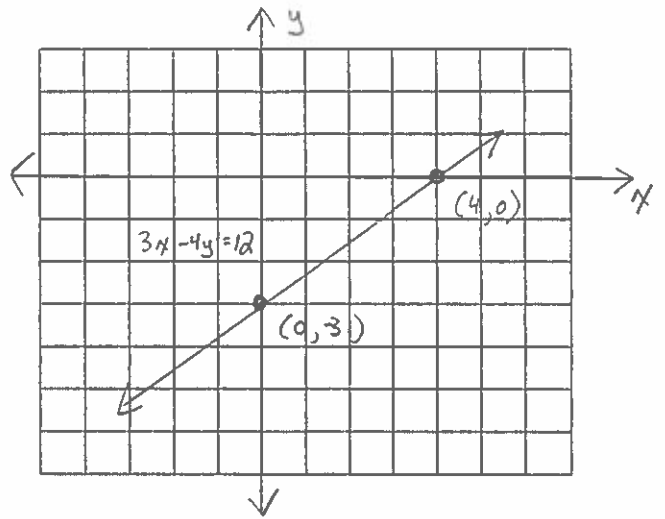
3.  $x+2y=-8$

$x\text{-int: } y=0$	$y\text{-int: } x=0$
$x=-8$	$2y=-8$
	$y=-4$
$(-8,0)$	$(0,-4)$



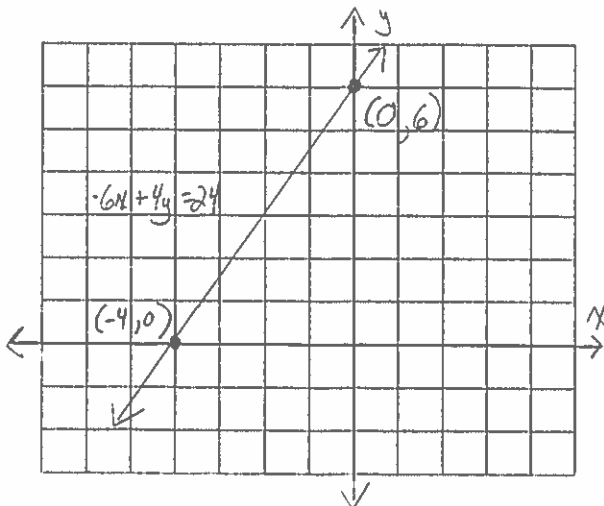
4.  $3x-4y=12$

$x\text{-int: } y=0$	$y\text{-int: } x=0$
$3x=12$	$-4y=12$
$x=4$	$y=-3$
$(4,0)$	$(0,-3)$



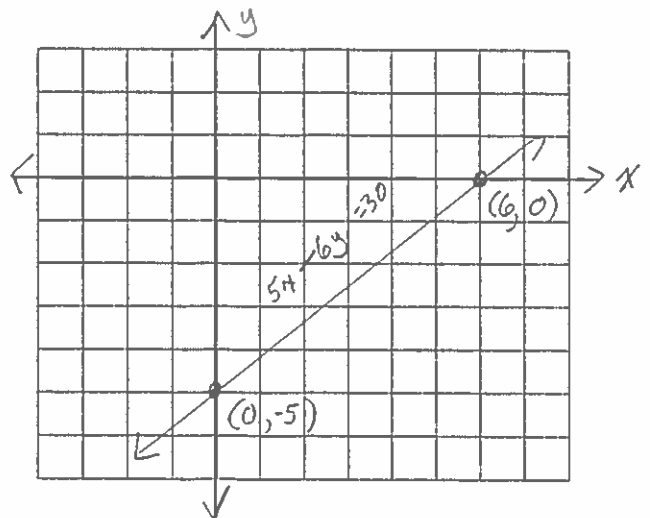
5.  $-6x+4y=24$

$x\text{-int: } y=0$	$y\text{-int: } x=0$
$-6x=24$	$4y=24$
$x=-4$	$y=6$
$(-4,0)$	$(0,6)$



6.  $5x-6y=30$

$x\text{-int: } y=0$	$y\text{-int: } x=0$
$5x=30$	$-6y=30$
$x=6$	$y=-5$
$(6,0)$	$(0,-5)$





## Section 4.4 Finding the Slope of a Line Using Two Points

**Ideas:** A line with positive slope rises from left to right. (Increasing Line)  
A line with negative slope falls from left to right. (Decreasing Line)  
A line with a zero slope does not change left to right. (Horizontal)  
A line with an undefined slope does not go left to right. (Vertical)

**Formula:** The slope of the line between two points can be calculated using the following formula.

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

**Example:** Find the slope of the line passing through the points (2, 1) and (4, 5). Describe the line as increasing, decreasing, horizontal, or vertical.

**Solution:** Let (2,1) be point 1 and (4, 5) be point 2.

$$m = \frac{\Delta y}{\Delta x} = \frac{5-1}{4-2} = \frac{4}{2} = 2$$

$m = 2$  The line is increasing.

**Example:** Find the slope of the line passing through the points (3, 3) and (3, -1). Describe the line as increasing, decreasing, horizontal, or vertical.

**Solution:** Let (3, 3) be point 1 and (3, -1) be point 2.

$$m = \frac{\Delta y}{\Delta x} = \frac{-1-3}{3-3} = \frac{-4}{0}$$

Slope is undefined. The line is vertical.

**Example:** Find the slope of the line passing through the points (2, -4) and (5, -4). Describe the line as increasing, decreasing, horizontal, or vertical.

**Solution:** Let (2, -4) be point 1 and (5, -4) be point 2.

$$m = \frac{\Delta y}{\Delta x} = \frac{-4 - (-4)}{5 - 2} = \frac{0}{3} = 0$$

$m = 0$  The line is horizontal.

### Practice for 4.4

A. Find the slope of the line passing through the given points.

B. Describe the line as increasing, decreasing, horizontal, or vertical.

1.  $(2, 3)$  and  $(4, 5)$

$$m = \frac{\Delta y}{\Delta x} = \frac{5-3}{4-2} = \frac{2}{2}$$

$m=1$     Inc. Line

2.  $(-2, 5)$  and  $(2, -3)$

$$m = \frac{\Delta y}{\Delta x} = \frac{-3-5}{2-(-2)}$$

$$m = \frac{-8}{4}$$

$m=-2$     Dec. Line

3.  $(3, 4)$  and  $(4, 4)$

$$m = \frac{\Delta y}{\Delta x} = \frac{4-4}{4-3}$$

$$m = \frac{0}{1}$$

$m=0$     Horizontal Line

4.  $(-7, 10)$  and  $(3, 0)$

$$m = \frac{\Delta y}{\Delta x} = \frac{0-10}{3-(-7)}$$

$$m = \frac{-10}{10}$$

$m=-1$     Dec. Line

5.  $(0, 4)$  and  $(0, -4)$

$$m = \frac{\Delta y}{\Delta x} = \frac{-4-4}{0-0}$$

$$m = \frac{-8}{0}$$

$m$  is undefined    Vertical Line

6.  $(2, -5)$  and  $(0, -4)$

$$m = \frac{\Delta y}{\Delta x} = \frac{-4-(-5)}{0-2}$$

$$m = \frac{1}{-2}$$

$m = -\frac{1}{2}$     Dec. Line

7.  $(9, 10)$  and  $(-5, 38)$

$$m = \frac{\Delta y}{\Delta x} = \frac{38-10}{-5-9}$$

$$m = \frac{28}{-14}$$

$m=-2$     Dec. Line

8.  $(-3, -2)$  and  $(3, -2)$

$$m = \frac{\Delta y}{\Delta x} = \frac{-2-(-2)}{3-(-3)}$$

$$m = \frac{0}{6}$$

$m=0$     Horizontal Line

## Section 4.5 Graphing Using Slope-Intercept Form

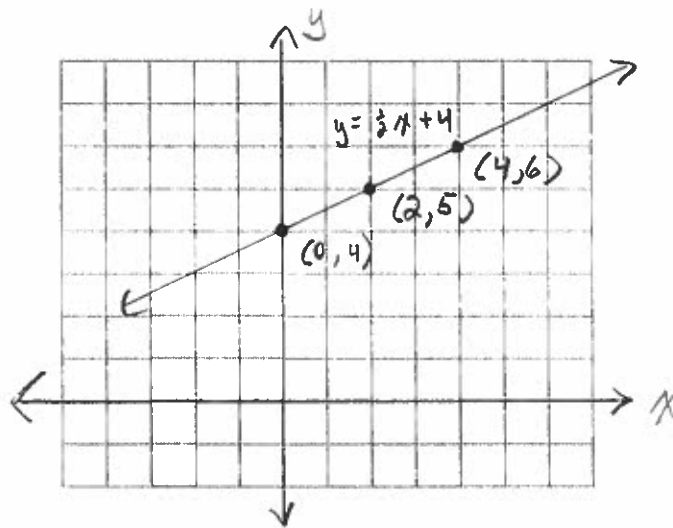
- Ideas:
1. Write  $y$  as a function of  $x$ .  $y = mx + b$   
 $m$  is the slope of the line.  $b$  is the  $y$ -coordinate of the  $y$ -intercept.  $(0, b)$
  2. Plot the  $y$ -intercept and use the slope to get 2 other points.
  3. Completely label your graph.

Example: Graph  $-x + 2y = 8$ .

$$2y = x + 8$$

$$y = \frac{1}{2}x + 4$$

The slope is  $\frac{1}{2}$  and the  $y$ -intercept is  $(0, 4)$ .



Practice for 4.5. Graph each equation using the slope and  $y$ -intercept.

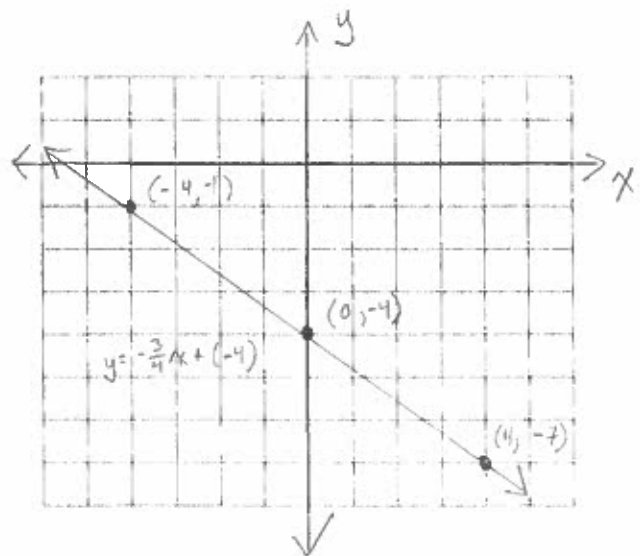
1.  $3x + 4y = -16$

$$4y = -3x + (-16)$$

$$y = -\frac{3}{4}x + (-4)$$

$$m = -\frac{3}{4} = \frac{\Delta y}{\Delta x} = \frac{-3}{4} = -\frac{3}{4}$$

$$y\text{-int: } (0, -4)$$



2.

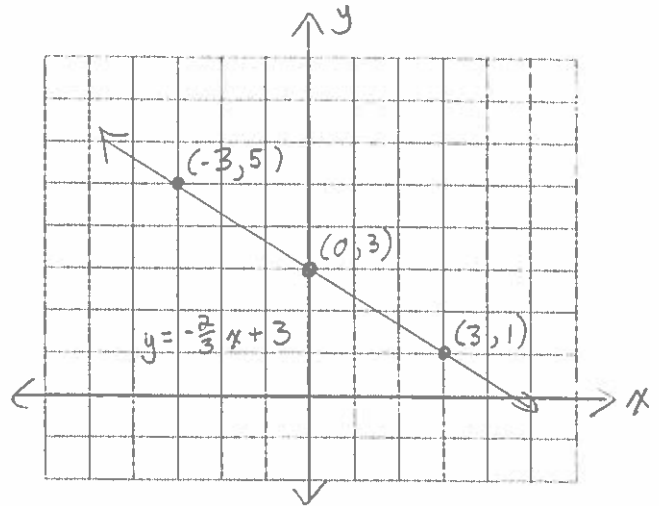
$$2x + 3y = 9$$

$$3y = -2x + 9$$

$$y = -\frac{2}{3}x + 3$$

$$m = -\frac{2}{3} = \frac{\Delta y}{\Delta x} = \frac{-2}{3} = -\frac{2}{3}$$

$$y\text{-int: } (0, 3)$$



3.

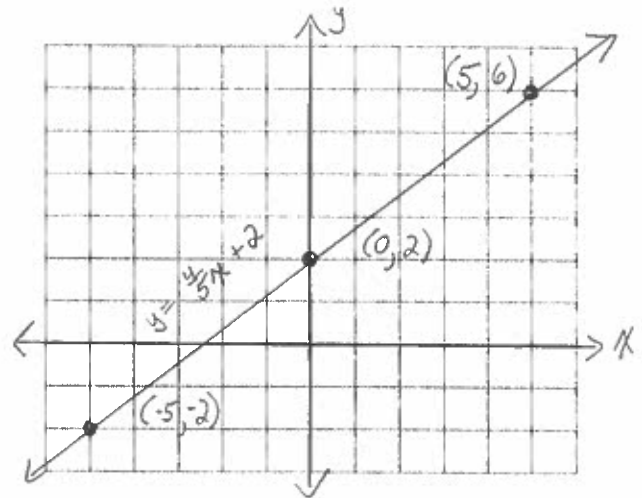
$$4x - 5y = -10$$

$$-5y = -4x + (-10)$$

$$y = \frac{4}{5}x + 2$$

$$m = \frac{4}{5} = \frac{\Delta y}{\Delta x} = \frac{-4}{-5}$$

$$y\text{-int: } (0, 2)$$



4.

$$-6x - 2y = 4$$

$$-2y = 6x + 4$$

$$y = -3x + (-2)$$

$$m = -3 = \frac{\Delta y}{\Delta x} = \frac{-3}{1} = -3$$

$$y\text{-int: } (0, -2)$$

